MODULE 11: Probability

Do revision of module 13 in the grade 11 X-Factor.

A. Singel Events

Probability (P): the chance that an event will occur
Sample Space (S): set of all possible outcomes

n**(S): number** of events in the sample space.

Probability of a single event E:

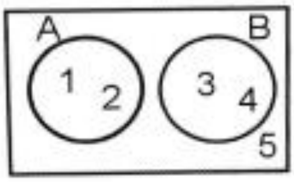
$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

B. More than one event

(A ∩ B) – intersection or (A and B): outcomes which occur in both A and B.

(A ∪ B) – union or (A or B): outcomes which occur in A or B.

1. Mutually exclusive

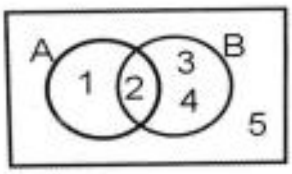


Events cannot take place simultaneously.

P(A and B) = 0

P(A or B) = P(A) +P(B)

2. Not mutually exclusive (inclusive)

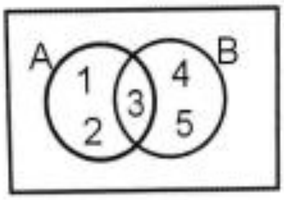


Events can take place simultaneously.

P(A and B) ≠ 0

P(A or B) = P(A) + P(B) – P(A ∩ B)

3. Exhaustive events

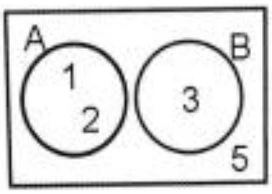


A and B contain all the elements of the sample space.

P(A ∪ B) = 1

Exhaustive events can have an intersection.

4. Complementary events



A' – complement of A.

A and A' are mutually exclusive AND exhaustive.

P(A') = 1 – P(A)

A and B mutually exclusive but not complementary.

Complement of A also includes the number 5. **A' = {3;5}**

C. <u>Independent events (Product-rule)</u>

The outcome of one event will **NOT** have an influence on the outcome of another event.

P(A and B) = P(A) x P(B)

D. <u>Not- independent events</u>

The outcome of one event will have an effect on the outcome of the other event.

P(A and B) ≠ P(A) x P(B)

P(A and B) = P(A) x P(B/A)   [B/A ....B follows A]

(e.g. taking candy from a bag without replacing it.)

Do C and D by using tree diagrams.

E. <u>The Fundamental Counting Principal</u>

If one event can happen in m ways and another event can happen in n ways, then both events can happen in m **x** n ways.

(when coin and dice are cast : 2 x 6 = 12 outcomes)

Examples

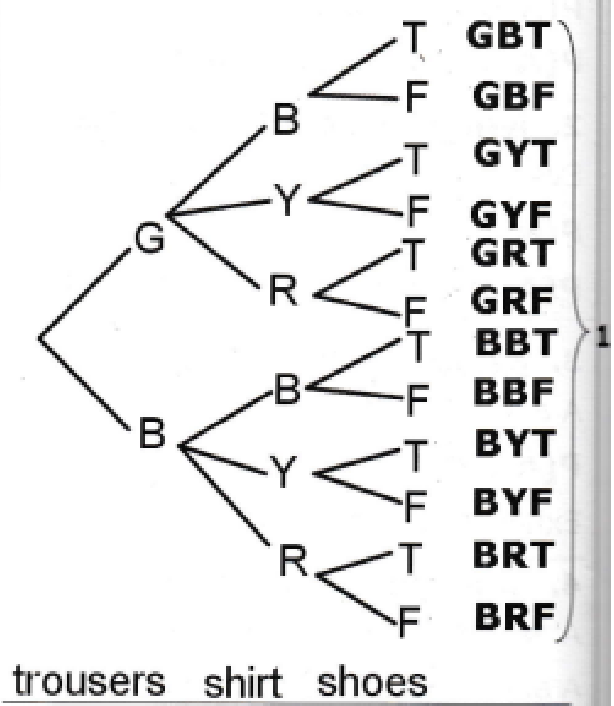
Question

How many different outfits are possible with the following combination of clothes?

Trousers	Shirts	Shoes
Grey Black	Blue yellow Red	Tekkies Flip-flops

Answer

- **With a tree diagram:**



- **With fundamental counting Rule:**

2 x 3 x 2 = 12 options

Trousers shirts shoes

This method much quicker!

F. Factorial ($n!$)

The product 7 x 6 x 5 x 4 x 3 x 2 x 1 can be written as 7!.
(read: seven factorial)

$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1$ Note: **0! = 1**

Factorial notation is used for the number of arrangements possible of one group of distinguishable objects.

$n!$ or $x!$ – key on calculator

Example	Answer
In how many ways can **5** people be arranged in a straight **line**?	5! = 5 x 4 x 3 x 2 x 1= 120 ways

G. Round-table arrangements ($(n-1)!$)

Example	Answer
In how many ways can **8** people be seated around an eight-seater **round table**?	(8 − 1)! = 7! = 5040 ways

H. Repition of objects $\frac{n!}{p!}$

The number of ways n objects of which p are identical, can be arranged is $\frac{n!}{p!}$

Example	Answer
(a) How many letter arrangements are possible with the word **BAFANA** ?	$\frac{6!}{3!} = 120$ 6 letters , 3 A^s
(b) How many letter arrangements can be made with the word **EENDEDAM**?	$\frac{8!}{3!.2!} = 3360$ 8 letters, 3 E^s , 2 D^s

I. Permutations ^nP_r

The number of arrangements of r objects from a total of n objects in a definite order is: $^nP_r = \frac{n!}{(n-r)!}$

Example	Answer
In how many ways can a chairperson, vice chairperson and secretary be selected from a group of **10** people?	$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$ $= \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1}$ $= 720$ **calculator: 10 / nPr / 3 = 720**

J. Probability

It is still defined by the **number of total possible outcomes devide by the number of total possibilities**

Example	Answer
1(a) Determine the probability that, in the letter arrangements of the word , **BAFANA**, it will start and end with the letter **A**.	**Total number of arrangements:** $= \frac{6!}{3!}$ = **120** **Number of outcomes** that start and end with **A**: **A** _ _ _ _ **A** = 1 x 4! x 1 = **24** (4 different remaining letters) ∴ **P**(start and end with A) $= \frac{24}{120} = 0{,}2$
1(b) How many combinations of meals are possible when the meal is made up of a starter course, a main course and pudding. There are 5 different starters, 7 main courses and 4 puddings to choose from.	5 x 7 x 4 = 140 possibilities
2. (a) In how many ways can the letters of the word **AANDAGTIG** be arranged? (Words do not need to have meaning.)	(a) $\frac{9!}{3!.2!}$ = **30240**
(b) What is the probability that, out of all arrangements, you will find one that starts and ends with the same letter?	(b) **Begins and ends with an A**: A _ _ _ _ _ _ _ A 7 remaining letters, 2 G^s^ $\frac{1 \times 7! \times 1}{2!}$ = **2520** **Begins and ends with a G:** G _ _ _ _ _ _ _ G 7 remaining letters, 3 A^s^ $\frac{1 \times 7! \times 1}{3!}$ = **840** **P**(begins and ends with same letter) $= \frac{2520 + 840}{30\ 240} = 0{,}\dot{1} = \frac{1}{9}$

Example	Answer
3. In how many ways can 3 boys and 2 girls be seated in the theater if:	
(a) anyone can be seated next to anyone?	(a) 5 children $\therefore 5! = 120$ ways
(b) the boys have to sit next to each other and the girls also have to sit next to one another?	(b) 3 boys - $3!$ 2 girls- $2!$ 2 groups - $2!$ $\therefore 3! \times 2! \times 2! = 24$ ways
4. (a) How many different codes can be generated if the following principles apply? • codes consist of 5 characters namely 2 letters and then 3 numbers. • Letters may be repeated, but not numbers. • The number, 0, may not be used.	(a) L L N N N L = 26 N = 9 $26 \times 26 \times 9 \times 8 \times 7 = 340\ 704$ Options
(b) If the above mentioned principles apply, determine the probability that a code will start with the letter K, and ends with an even number.	(b) K ☐ ☐ ☐ ☐ (2, 4, 6, 8) Number of options $= 1 \times 26 \times 8 \times 7 \times 4 = 5824$ $\therefore$ **P**(starts with K, ends with even number) $= \frac{5824}{340\ 704} = 0{,}017$
5(a) In how many ways can **ABC** be arranged in groups of 2 letters if order does matter?	(a) **AB...BA...CB...BC...AC...CA** $3! = 3 \times 2 \times 1 = 6$
(b) If **order does not matter.**	(b) **AB...CB...AC (divide by 2!)** $\frac{3!}{2!} = \frac{6}{2} = 3$

Worksheet 11 A (Revision)

1. A and C are mutually exclusive events. If P(A) = 0,24 and P(A or C) = 0,43, determine P(C).

2. If P(A) = 0,36, P(B) = 0,24 and P(A or B) = 0,52, determine:

2.1 P(A and B)

2.2 P(A')

3. In an experiment it was found that P(A) = 0,6, P(B) = 0,35 and P(A or B) = 0,75. Determine:

3.1 P(A and B).

3.2 Determine whether the events A and B are independent.(show calculations)

3.3 Determine, stating reasons whether A and B are mutually exclusive.

3.4 Why will events A and B NOT be complementary events?

4. For a certain sample set, the following results were measured regarding events K and L :

P(K) = 0,45, P(K or L) = 0,74, P(L) = x.

4.1 For which value(s) of x will K and L be mutually exclusive?

4.2 For which value(s) of x will K and L be independent?

5. The probability that a person is bilingual is 0,65. The probability that a person has a musical talent is 0,15. Determine the probability that:

5.1 a person is bilingual and has a musical talent if the two events are independent.

5.2 a person has none of the two characteristics.

5.3 a person is either bilingual or has a musical talent.

Worksheet 11 B (Tree diagram)

1. Two coins are cast. One of the coins has been weighed to one side, such that the probability to have heads is, $\frac{4}{7}$. The other coin is just a normal coin.

1.1 Draw a tree diagram to show all possible outcomes.

1.2 Will the casting of the two coins be dependent or independent? (Motivate)

1.3 Determine the probability of getting heads on both coins.

2. A bag contains 6 white marbles and 4 blue marbles. Three marbles are taken consecutively out of the bag. Each time that a white marble is found, it is put back in the bag. When a blue marble is selected, it is Not put back.

2.1 Draw a tree diagram to show all possible outcomes.

2.2 Calculate the following probabilities:

2.2.1 that all 3 marbles selected are blue

2.2.2 P(two blue marbles)

2.2.3 P(at least one of the marbles selected is blue)

3. The following probabilities regarding a certain race-horse, Lightning, were calculated from past experience during the Durban July racehorsing events:

The probability that Lightning win in dry weather is three times more than in wet weather. The probability that Lightning will win in wet weather was estimated as 0,31. Die probability that it will rain on any given day of the Durban July is 35%. Calculate the probability that Lightning will win. (Use a tree diagram)

Worksheet 11 D

3. A car-insurance company is interested in the relationship between the age of drivers and the number of accidents that they have made over a period of 10 years.

   The results of a survey of 1000 drivers is shown in the two-way table below:

Age of driver	Number of accidents		Tot
	5 or less	More than5	
28 years or older	(a)	125	(e)
Younger than 28 years	500	(b)	750
Total	(c)	(d)	(f)

3.1 Calculate the values of (a) to (f). (Not necessarily in this particular order)

3.2 Determine whether the number of accidents made by a person, is independent or not of that person's age. (show calculations to explain your answer)

Worksheet 11 E
(Fundamental Counting Principal)

1.1 How many different outfits are possible from the items in my closet?

Trousers	Shirts	Shoes
Blue	White	Black
Brown	Blue	Brown
Black	Red	
	Green	
	Purple	

1.2 What is the probability that a person will walk out of his house wearing a blue shirt?

2. At a popular restaurant people can choose from a set menu to make up their meals. Sherry is served at the start and coffee or tea at the end of the meal. A person may choose one starter, one main course and one of the puddings on the menu.

Worksheet 11 E

2.

Starters	Main	Pudding
Soup	Chicken pie	Chocolate mousse
Fish	Venison	Ice cream
Salad	Lamb shank	Tiramisu
Cray-fish	Pork Ribs	Cheese cake
	Beef fillet	Malva-pudding
	Bobotie	
	Chicken a la King	

2.1 In how many ways can a person choose a meal at this reastaurant?

2.2 In how many ways can a meal be made up if a person does not eat chicken and he/she is allergic to cray-fish? (the person may still only select one of each course)

3. In how many ways can 5 different books be arranged on a book shelf?

4. In how many ways can 8 learners arrange themselves in a line?

5. In how many ways can six people be seated at a round table?

6. In how many ways can 3 English and 5 Afrikaans novels be arranged on a shelve if:

6.1 any book can take up any position

6.2 if the Afrikaans books and the English books must be arranged together in any order?

7. A library buys an encyclopedic set, consisting of 12 volumes. The books are numbered from 1 to 12.

7.1 In how many ways can the books be arranged if the books can be placed in any order?

7.2 In how many ways can the books be arranged if number 1 and number 12 must be the first and last books respectively.(the other books may be arranged in any order)

Worksheet 11 E

7.3 In how many ways can the books be arranged if numbers 9, 10 and 11 should always be next to one another, but in any order?

Worksheet 11 F (FCP)

1. Given the numbers:

   1, 2, 3, 4, 5, 6, 7, 8, 9

1.1 How many two-digit numbers can be formed using the numbers above? (numbers may be used repeatedly.)

1.2 How many 4-digit pin codes can be formed using the numbers above? (numbers may only be used once)

1.3 How many 4-digit pin codes are possible if the code has to start with 3 and numbers may be used repeatedly?

1.4 How many 4-digit codes are possible if the code has to start with 1 and end with 9? (Numbers may not be used more than once)

1.5 How many 4-digit codes are possible if the code should at least have one 5 and numbers may be repeated?

2. How many 3-character codes can be formed if the first character should be a letter and the next two characters have to be numbers. All the characters should differ from one another.

3. Given the numbers:

   2, 3, 4, 5, 6, and 7

   How many 5-digit codes can be formed with these numbers if:

3.1 numbers may be repeated?

3.2 numbers may not be repeated?

3.3 How many 3-digit codes are possible if numbers may be repeated and the code that is formed should be greater than 400 and less than 600 and divisible by 5?

Worksheet 11 F (FCP)

4. How many even 4-digit codes are possible when using the numbers 0, 1, 2, 3, 4 and 5? Codes may not start with 0 and numbers may be repeated.

5. Passwords for a certain website can be chosen as follows:

   The password must consists of at least 6 characters and at most 8 characters.
   Letters from which to choose: A – L
   Numbers from which to choose: 0 – 9
   Symbols: **# and ***
   Passwords should start with 3 letters, then numbers and end with one of the symbols.

5.1 How many 7-character passwords are possible if characters may not be used more than once?

5.2 How many 7-character passwords are possible if letters may not be repeated, but numbers may be used more than once?

5.3 What is the probability that a 7- character password will start with ABC in the letter part? (no repetition allowed)

5.4 What is the probability that a 7- character password code has exactly one 2? (repetition not allowed)

5.5 If repetition of characters is NOT allowed, calculate the probability that a website user has a password consisiting of 6 characters.

Worksheet 11 F

5.6 The website changes the format of the passwords and decided that any character may be chosen at any position. Repetition of characters is allowed. Determine the probability that a randomly chosen person will have a 7-character password.

Worksheet 11 G

1.1 How many different "words" can be formed using the letters of the word **GEOMETRY**?

1.2 Assuming that the word has to begin with R, in how many ways can the letters be arranged to form words?

1.3 Assuming that the word has to begin with G and end with Y, how many possible "words" can be formed?

1.4 What is the probability that an arrangement chosen randomly, will begin with G and end with Y?

2.1 In how many ways can the letters of the word **KAAPSTAD** , be arranged to form words?

2.2 What is the probability that one of the "words", if randomly chosen from the pool of all possible words will:

2.2.1 begin with D

2.2.2 begin with A

2.2.3 begin and end with A.

3.1 How many 6-letter combinations of "words" can be formed with letters A, B, K, D, E, T if each letter may only be used once?

3.2 Assuming that the probability to have a meaningful word from all the possible combinations is 0,05, how many of the possibilities will be meaningful?

Worksheet 11 G

4. Given the word **MISSISIPPI**. What is the probability that a random selection of a letter combination made with the letters, will begin and end with the same letter?

5. Mari randomly chooses 3 numbers from the numbers 2 to 8. She asks Gerhard to guess the numbers chosen. What is the probability that he will guess the numbers correctly the first time? It does not matter in which order the numbers are given.

6. In a lotto-game, 6 numbers are selected from the numbers 1 to 49. In how many ways can the numbers be selected if:

6.1 selection of the numbers happens without returning the selected number to the pool of numbers and the order in which they are selected is important to be able to win.

6.2 selection of the numbers happens without returning the selected number, but the order in which they are drawn, is not important to win.

6.3 What is the probability for a person who bought one lotto ticket to win the lotto, if the procedure is done as described in 6.2?

6.4 Henro decides to buy tickets for all the possible outcomes determined in 6.2 to ensure that he wins the LOTTO-jackpot. If the cost of a lotto ticket is R3,50 each, how much money will he have to spend?